Linear Algebraic Semantics for Natural Language

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Mathematical Approaches to Natural Language

Grammar

Set-Theoretic

Pragmatics

Mathematical Approaches to Natural Language

Grammar

Chomsky

Context Free Grammars

S -> NP VP

NP -> cats

VP -> sneeze

S -> NP VP

-> cats VP

-> cats sneeze

Mathematical Approaches to Natural Language

Grammar

Ajdukiewic, Lambek

Type Algebras

cat: **NP**

sneeze: **NP^r S**

cats sneeze

NP (NP^r S) → S

Mathematical Approaches to Natural Language

Set-Theoretic

Montague

First Order Logic

cats sneeze $\exists x \; cat(x) \wedge sneeze(x)$

Mathematical Approaches to Natural Language

Pragmatics

Harris, Firth, Saussure

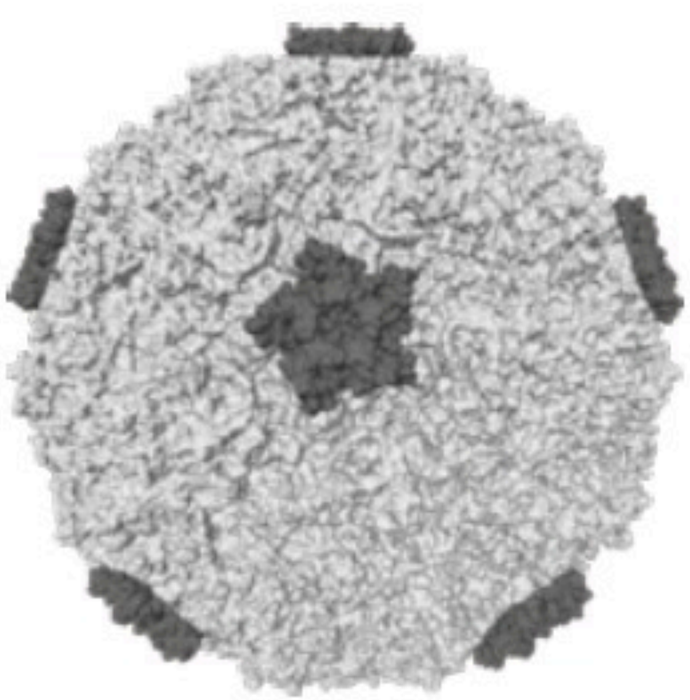
Vector Space Semantics

Meaning of a word depends on its use in the context.

Words that have similar meanings often occur in the same context.

Guessing Meaning from Context

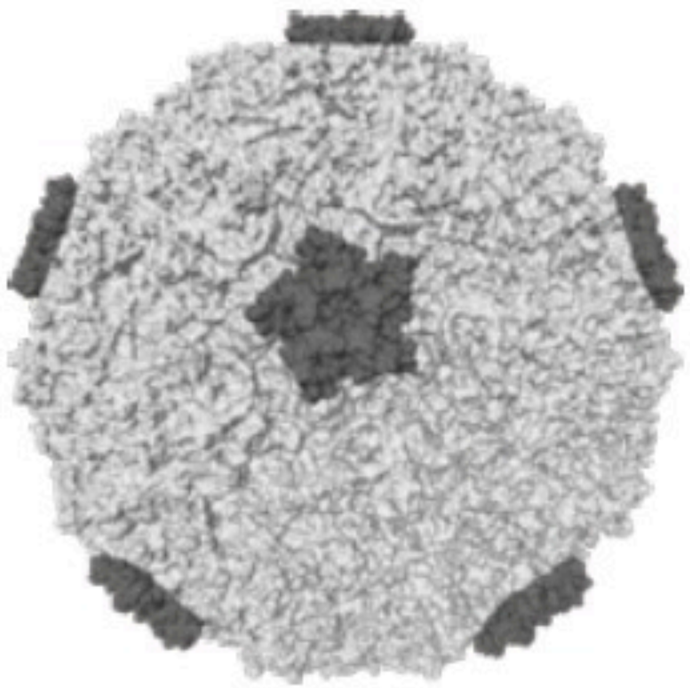
مقالهٔ برگزیده



______ یک بیماری واگیردار مربوط به دستگاه تنفسی فوقانی است که عمدتاً بینی را تحت تأثیر قرار می‌دهد. ______ معمولاً با خستگی، احساس سرما، عطسه و سردرد آغاز می‌شود و با علائمی چون سرفه، گلودرد، آبریزش بینی و تب ادامه می‌یابد و معمولاً هفت تا ده روز بعد برطرف می‌شود و برخی علائم ممکن است تا سه هفته طول بکشد. بیش از دویست نوع ویروس عامل ______ وجود دارد، با این حال راینو ویروس‌ها (که خود بیش از ۹۹ نوع مختلف شناخته‌شده هستند) متداول‌ترین عامل این بیماری هستند. ویروس‌های عامل بیماری می‌توانند تا مدت زمانی طولانی (برای راینو ویروس تا بیش از ۱۸ ساعت) در محیط زنده بمانند و ممکن است از دستان به چشمان و بینی که محل عفونت هستند، منتقل شوند. ویروس از طریق عطسه، سرفه و تماس با افراد یا اشیاء آلوده قابل انتقال به بدن است. برخی معتقدند که در معرض سرما قرار گرفتن باعث ______ می‌شود و همین باعث شده که این نام را برای بیماری انتخاب کنند که البته این مطلب به اثبات نرسیده و رد شده است.

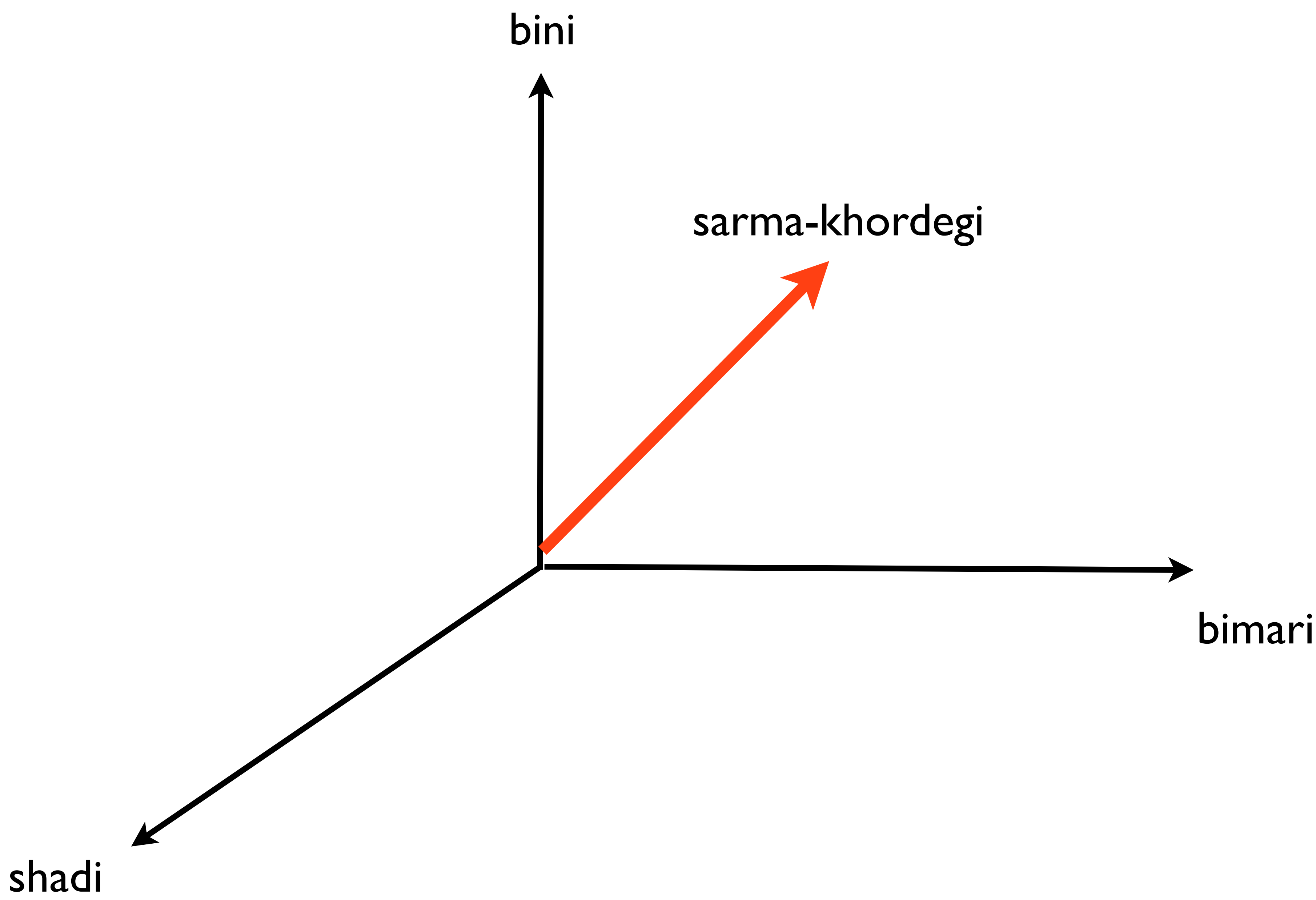
Guessing Words from Context

مقالهٔ برگزیده

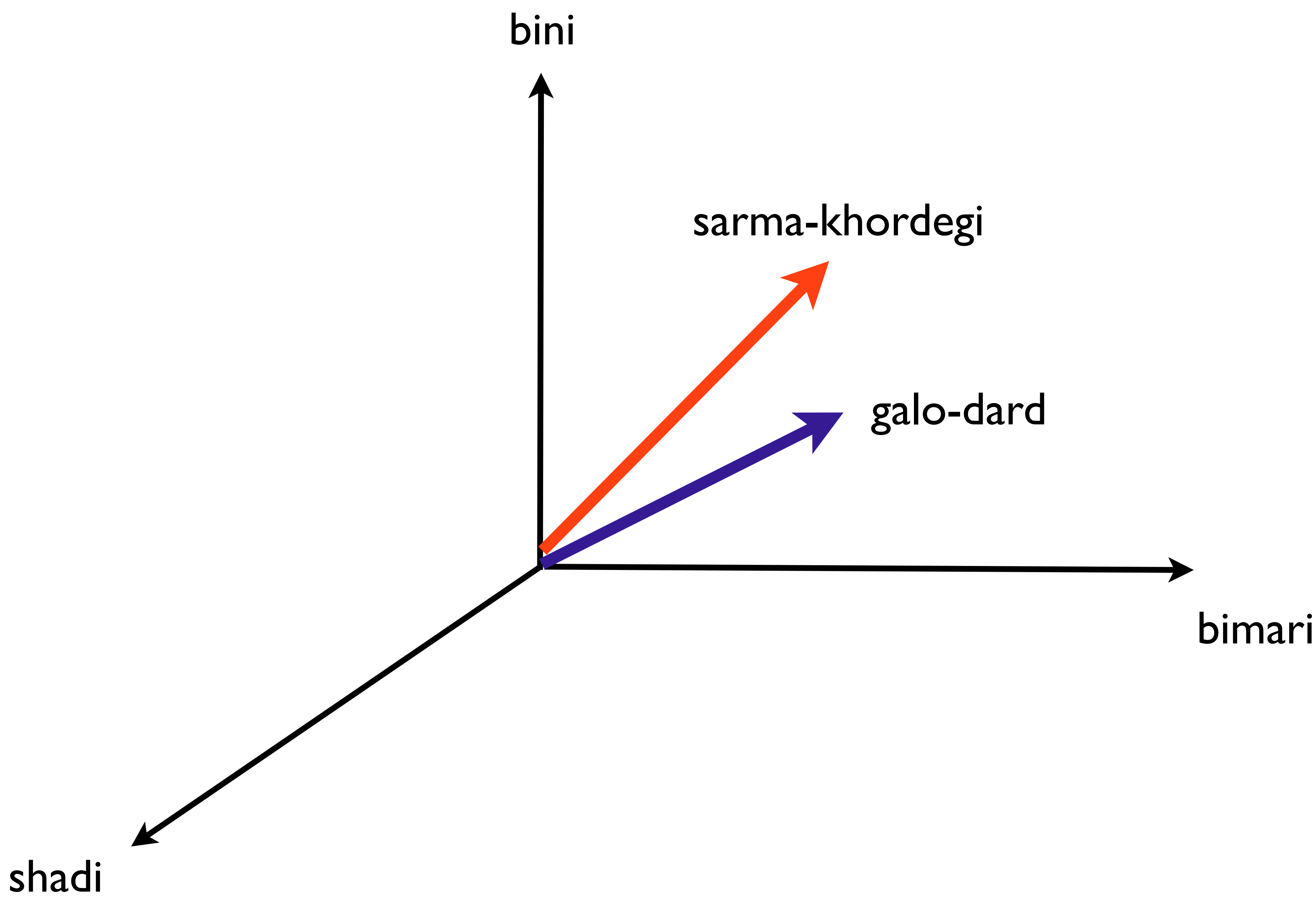


سرماخوردگی یک بیماری واگیردار مربوط به دستگاه تنفسی فوقانی است که عمدتاً بینی را تحت تأثیر قرار می‌دهد. سرماخوردگی معمولاً با خستگی، احساس سرما، عطسه و سردرد آغاز می‌شود و با علائمی چون سرفه، گلودرد، آبریزش بینی و تب ادامه می‌یابد و معمولاً هفت تا ده روز بعد برطرف می‌شود و برخی علائم ممکن است تا سه هفته طول بکشد. بیش از دویست نوع ویروس عامل سرماخوردگی وجود دارد، با این حال راینو ویروس‌ها (که خود بیش از ۹۹ نوع مختلف شناخته‌شده هستند) متدوال‌ترین عامل این بیماری هستند. ویروس‌های عامل بیماری می‌توانند تا مدت زمانی طولانی (برای راینو ویروس تا بیش از ۱۸ ساعت) در محیط زنده بمانند و ممکن است از دستان به چشمان و بینی که محل عفونت هستند، منتقل شوند. ویروس از طریق عطسه، سرفه و تماس با افراد یا اشیاء آلوده قابل انتقال به بدن است. برخی معتقدند که در معرض سرما قرار گرفتن باعث سرماخوردگی می‌شود و همین باعث شده که این نام را برای بیماری انتخاب کنند که البته این مطلب به اثبات نرسیده و رد شده است.

Vector Representations for Words



Vector Representations for Words



Problems with each approach

Grammar

Spurious Ambiguity *I saw the man with a telescope.*

Set-Theoretic

Scope and Binding *If a man has a cat, he feeds it.*

$$\forall x(man(x) \wedge \exists y\ cat(y) \wedge has(x,y)) \implies feeds(x,y)$$

Modularity cats sneeze $\forall x\ cat(x) \implies sneeze(x)$

Problems with each approach

Grammar

Spurious Ambiguity

Set-Theoretic

Scope and Binding
Modularity

⇨

Single words have no meaning.

Problems with each approach

Grammar

Spurious Ambiguity

Set-Theoretic

Scope and Binding
Modularity

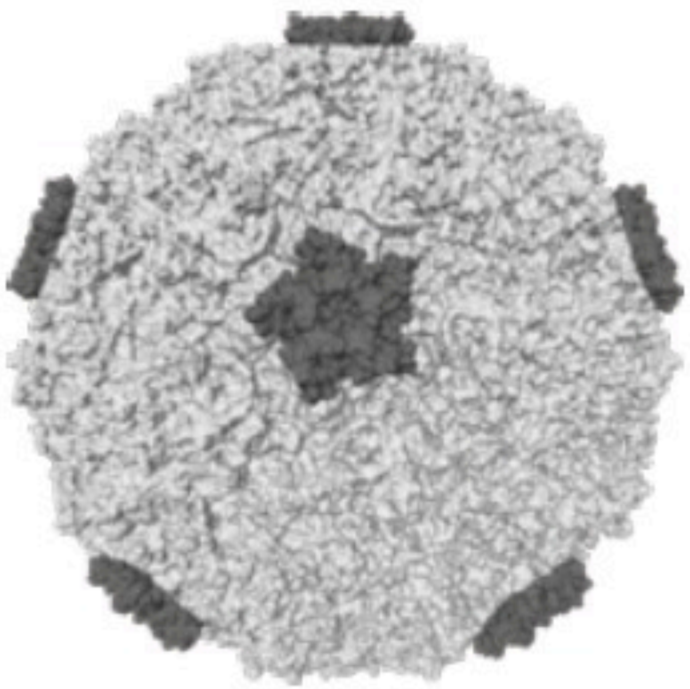
Pragmatics

Grammatical structure is not represented.

→ ***Single words have no meaning.***

Guessing Sentences from Context

مقالهٔ برگزیده



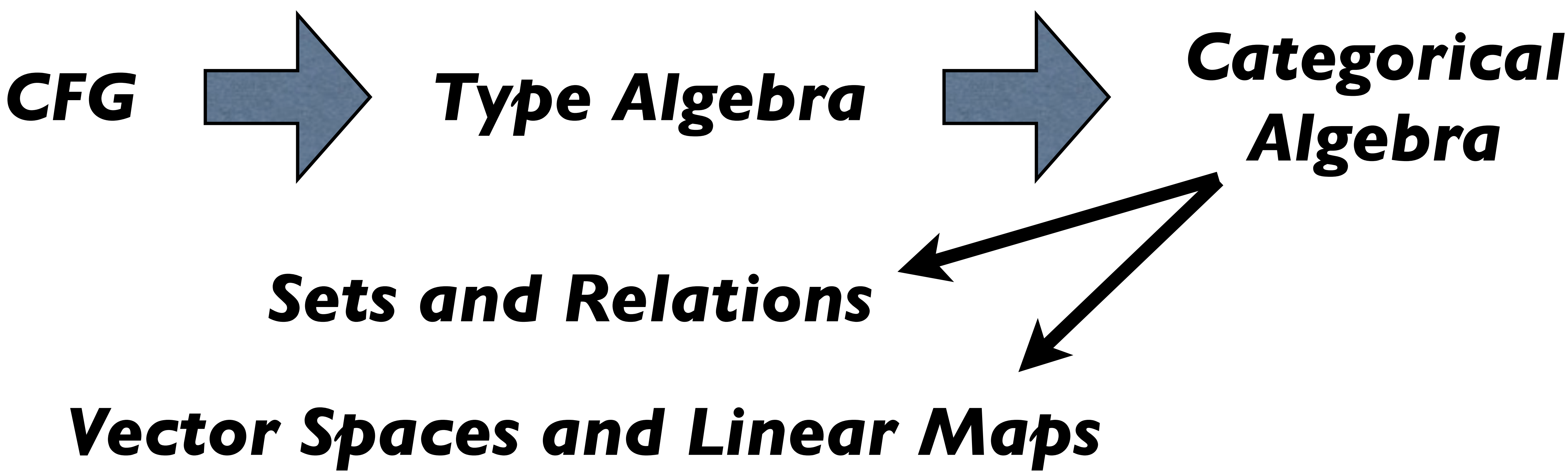
[redacted]

سرماخوردگی معمولاً با خستگی، احساس سرما، عطسه و سردرد آغاز می‌شود و با علائمی چون سرفه، گلودرد، آبریزش بینی و تب ادامه می‌یابد و معمولاً هفت تا ده روز بعد برطرف می‌شود و برخی علائم ممکن است تا سه هفته طول بکشد. بیش از دویست نوع ویروس عامل سرماخوردگی وجود دارد، با این حال راینو ویروس‌ها (که خود بیش از ۹۹ نوع مختلف شناخته‌شده هستند) متدوال‌ترین عامل این بیماری هستند. ویروس‌های عامل بیماری می‌توانند تا مدت زمانی طولانی (برای راینو ویروس تا بیش از ۱۸ ساعت) در محیط زنده بمانند و ممکن است از دستان به چشمان و بینی که محل عفونت هستند، منتقل شوند. [redacted]

Goal

Develop a mathematical model of natural language that does not have (or has as little as possible of) (or less of) these problems.

Methodology



Context Free Grammar

$$G = (T, N, S, \mathcal{R})$$

Terminals: $a, b, c, \in T$

Non-terminals: $A, B, C \in N$

End symbol: $S \in N$

Chomsky Normal Form

$$A \to BC \qquad A \to a$$

Application

$$\beta A \gamma \xrightarrow{A \to \alpha} \beta \alpha \gamma$$

Generation from S ending in T: grammatical sentences

Generalised Quantifiers

Barwise and Cooper, 1981

Context Free Grammar $G = (T, N, S, \mathcal{R})$

$$\begin{aligned} \text{S} &\rightarrow \text{NP VP} \\ \text{VP} &\rightarrow \text{V NP} \\ \text{NP} &\rightarrow \text{Det N} \end{aligned}$$

$$\begin{aligned} \text{NP} &\rightarrow \text{John, Mary, something, } \cdots \\ \text{N} &\rightarrow \text{cat, dog, man, } \cdots \\ \text{VP} &\rightarrow \text{sneeze, sleep,} \cdots \\ \text{V} &\rightarrow \text{love, kiss, } \cdots \\ \text{Det} &\rightarrow \text{a, the, some, every, each, all, no, most, few, one, two, } \cdots \end{aligned}$$

Generalised Quantifiers

Barwise and Cooper, 1981

Set Theoretic Model $(U, [\![\ ]\!])$

Semantics of words

$$\begin{aligned}
\text{NP} \to np \quad &\Longrightarrow \quad [\![np]\!] \in U \\
\text{N} \to n \quad &\Longrightarrow \quad [\![n]\!] \subseteq U \\
\text{VP} \to vp \quad &\Longrightarrow \quad [\![vp]\!] \subseteq U \\
\text{V} \to v \quad &\Longrightarrow \quad [\![v]\!] \subseteq U \times U \\
\text{Det} \to d \quad &\Longrightarrow \quad [\![d]\!] : \mathcal{P}(U) \to \mathcal{P}\mathcal{P}(U)
\end{aligned}$$

Generalised Quantifiers

Barwise and Cooper, 1981

Semantics of determiners

$$[\![some]\!](A) = \{X \subseteq U \mid X \cap A \neq \emptyset\}$$

$$[\![\mathrm{E}very]\!](A) = \{X \subseteq U \mid A \subseteq X\}$$

$$[\![\mathrm{n}o]\!](A) = \{X \subseteq U \mid A \cap X = \emptyset\}$$

$$[\![n]\!](A) = \{X \subseteq U \mid \mid X \cap A \mid = n\}$$

$$[\![\mathrm{m}ost]\!](A) = \{X \subseteq U \mid X \text{ contains most } A\text{'s}\}$$

$$[\![\mathrm{f}ew]\!](A) = \{X \subseteq U \mid X \text{ contains few } A\text{'s}\}$$

Generalised Quantifiers

Barwise and Cooper, 1981

Semantics of phrases

- $A \to BC$ $\qquad \llbracket A \rrbracket := \llbracket C \rrbracket(\llbracket B \rrbracket)$

- $\llbracket \text{Det N} \rrbracket = \llbracket d \rrbracket(\llbracket n \rrbracket)$ where $X \in \llbracket d \rrbracket(\llbracket n \rrbracket)$ **iff** $(X \cap \llbracket n \rrbracket) \in \llbracket d \rrbracket(\llbracket n \rrbracket)$
  for $d \in Det, n \in N$

Living On

Truth Theoretic Semantics

Meaning of a sentence is true iff $[\![vp]\!]([\![np]\!]) = 1$

For instance

Sentence with quantified subject

Living On

$[\![\text{some men sneeze}]\!] = 1$ *iff* $[\![sneeze]\!] \cap [\![\text{m}en]\!] \in [\![\text{some men}]\!]$

Translate CFG to Type Algebra

Ajdukiewic, Bar-Hillel, Lambek

CFG → **RAG** → **LG** → **PRG**

Bar-Hillel Shamil Gaifan	Cohen Buszkowski	Buszkowski

CFG → **PRG**

Kuzenstov   Buszkowski

Type Algebra

$$P = (P, \Sigma, \beta, s)$$

Pregroup Algebra

$$P = (P, \leq, \cdot, (-)^r, (-)^l)$$

Vocab

Type Dictionary

$$\beta \subseteq \Sigma \times P$$

designated element

Application $\quad p \cdot p^r \leq 1 \qquad p^l \cdot p \leq 1$

Grammatical Reduction $\quad w_1 \cdots w_n$ **for** $w_i \in \Sigma$

$$p_1 \cdot \quad \cdots \quad \cdot p_n \leq p \qquad \textbf{for} \qquad p_i \in \beta(w_i)$$

Translate CFG to Type Algebra

Words

Non-terminal x	$\sigma(x)$
John, Mary, something, $\cdots$	p
cat, dog, man, $\cdots$	n
sneeze, sleep,$\cdots$	$p^r \cdot s$
love, kiss, $\cdots$	$p^r \cdot s \cdot p^l$
a, the, some, every, each, all, no, most, few, one, two, $\cdots$	$p \cdot n^l$

Reductions

$$\underset{(p \cdot n^l)}{\text{some}} \quad \underset{\cdot n}{\text{cats}} \qquad \leq p \cdot 1 = p$$

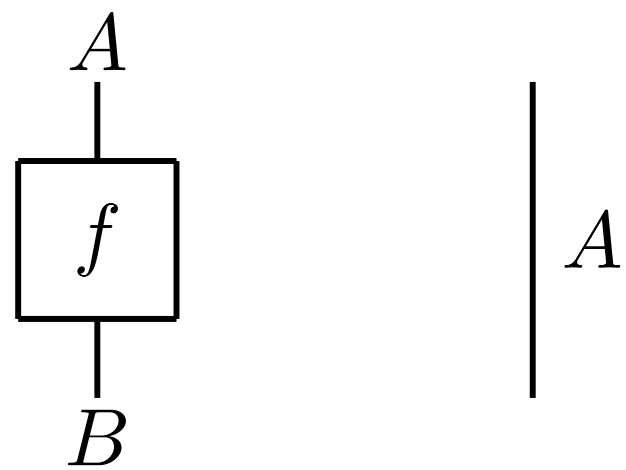
$$\underset{(p \cdot n^l)}{\text{some}} \quad \underset{\cdot n}{\text{cats}} \quad \underset{\cdot(p^r \cdot s)}{\text{sneeze}} \qquad \leq p \cdot 1 \cdot (p^r \cdot s) = p \cdot (p^r \cdot s) \leq 1 \cdot s = s$$

$$\underset{p}{\text{John}} \quad \underset{\cdot(p^r \cdot s \cdot p^l)}{\text{kissed}} \quad \underset{\cdot(p \cdot n^l)}{\text{some}} \quad \underset{\cdot n}{\text{cats}} \qquad \leq \quad 1 \cdot (s \cdot p^l) \cdot p \cdot 1 = (s \cdot p^l) \cdot p \leq s \cdot 1 = s$$

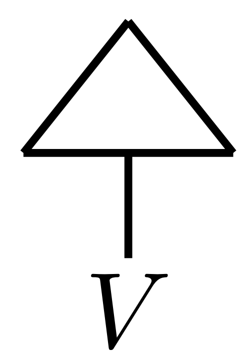
Compact Closed Categories

Kelly and Laplaza, 1980

Objects and morphisms



Elements within objects

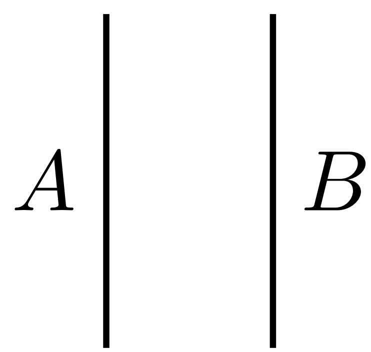


Compact Closed Categories

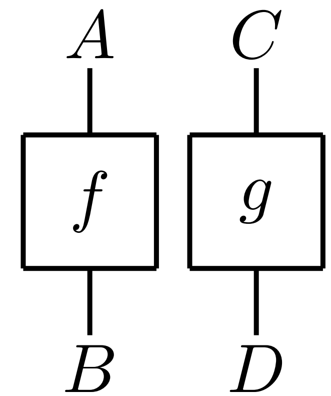
Kelly and Laplaza, 1980

Binary product of objects and morphisms

$A \otimes B$

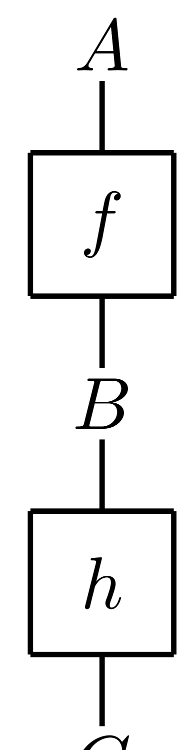


$f \otimes g$



Sequential composition of morphisms

$f \circ h$



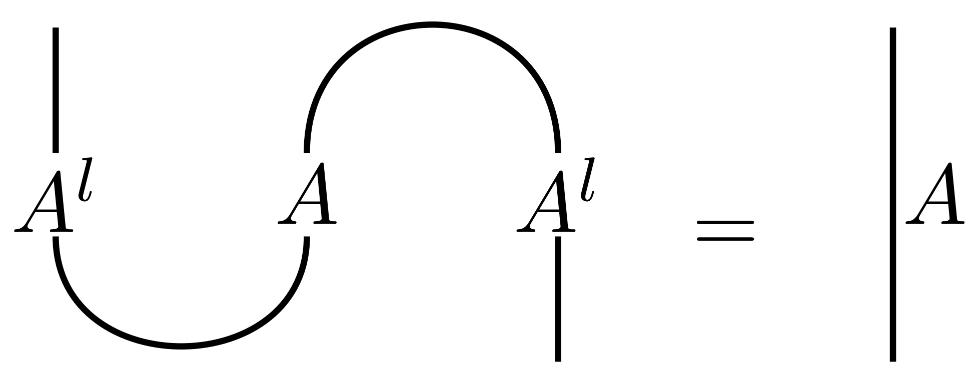
Compact Closed Categories

Kelly and Laplaza, 1980

Epsilon and eta maps

$$\epsilon^l \colon A^l \otimes A \to I, \eta \colon I \to A \otimes A^l$$

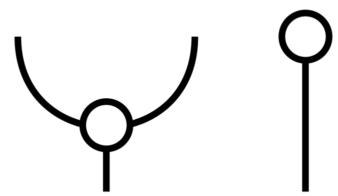
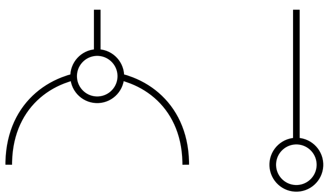
satisfying



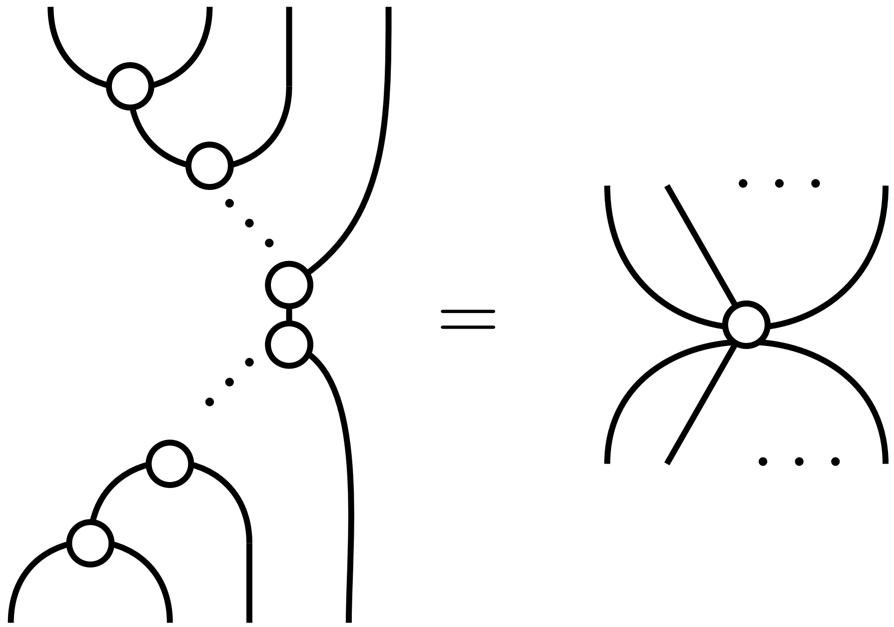
Frobenius Algebras

Kock, 1972

Combining and dispatching

(μ, ζ)  (Δ, ι) 

satisfying



Example: Vector Spaces

Objects: vector spaces **Elements**: vectors

Morphisms: linear maps **Product**: tensor product

Epsilon maps: dot product

$$\epsilon^l = \epsilon^r : V \otimes V \to \mathbb{R} \quad \text{given by} \quad \sum_{ij} c_{ij}\, \psi_i \otimes \phi_j \quad \mapsto \quad \sum_{ij} c_{ij} \langle \psi_i \mid \phi_j \rangle$$

Eta maps: diagonal matrix

$$\eta^l = \eta^r : \mathbb{R} \to V \otimes V \quad \text{given by} \quad 1 \mapsto \sum_i r_i \otimes r_i$$

Combining: sends a vector to a diagonal matrix

$$\delta : V \to V \otimes V \quad \text{given by} \quad \overrightarrow{v_i} \mapsto \overrightarrow{v_i} \otimes \overrightarrow{v_i}$$

$$\mu : V \otimes V \to V \quad \text{given by} \quad \overrightarrow{v_i} \otimes \overrightarrow{v_j} \mapsto \delta_{ij}\, \overrightarrow{v_i}$$

Example: Sets and Relations

Objects: sets

Elements: individuals

Morphisms: relations

Product: cartesian product

Epsilon and eta maps

$$\epsilon^l = \epsilon^r \; : S \times S \to \{\star\} \quad \text{given by} \quad \{((s_i, s_j), \star) \mid s_i, s_j \in S, s_i = s_j\}$$

$$\eta^l = \eta^r \; : \{\star\} \to S \times S \quad \text{given by} \quad \{(\star, (s_i, s_j)) \mid s_i, s_j \in S, s_i = s_j\}$$

Combining and dispatching

$$\delta \; : S \to S \times S \quad \text{given by} \quad \{(s_i, (s_j, s_k)) \mid s_i, s_j, s_k \in S, s_i = s_j = s_k\}$$

$$\mu \; : S \times S \to S \quad \text{given by} \quad \{(s_i, s_j), s_k) \mid s_i, s_j, s_k \in S, s_i = s_j = s_k\}$$

Categorical Semantics

Model $(\mathcal{C}, W, S, \overline{\llbracket\ \rrbracket})$

Compact Closed Category

Designated Objects with Frob. Structure

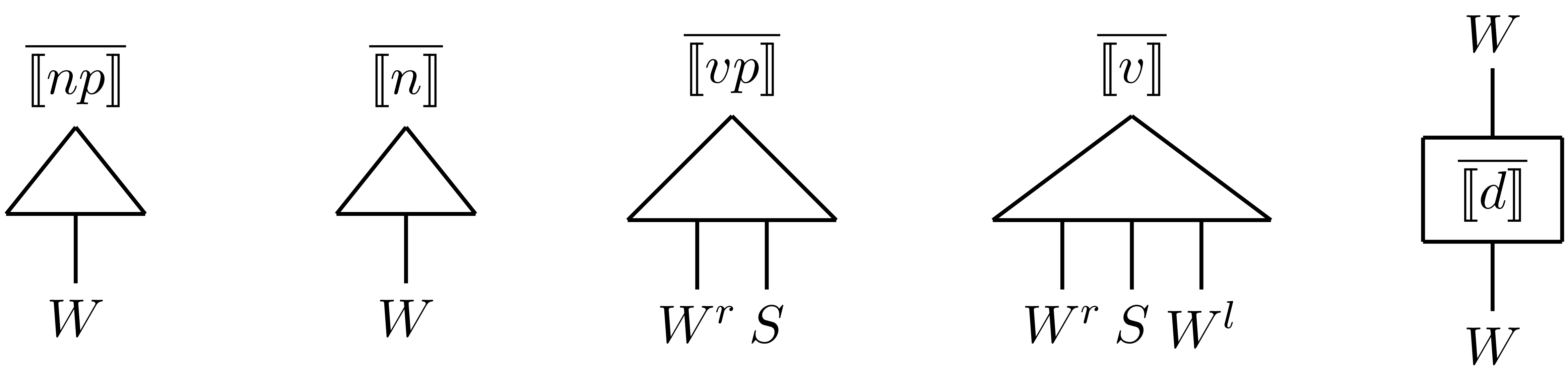
Semantics of types $\overline{\llbracket p \rrbracket} = \overline{\llbracket n \rrbracket} = W \qquad \overline{\llbracket s \rrbracket} = S$

Semantics of words $\overline{\llbracket w \rrbracket} := I \to \overline{\llbracket \sigma(w) \rrbracket}$

Semantics of strings $\overline{\llbracket w_1 \cdots w_n \rrbracket} := \overline{\llbracket \alpha \rrbracket}\Big(\overline{\llbracket w_1 \rrbracket} \otimes \cdots \otimes \overline{\llbracket w_n \rrbracket}\Big)$

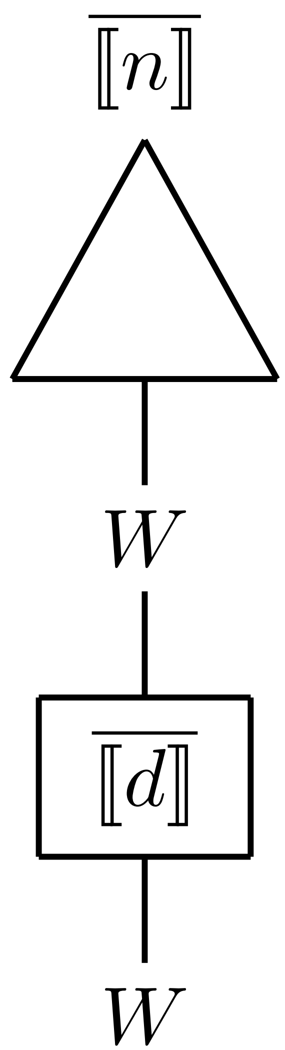
Categorical Algebra

Pictorially



Compact Closed Models of Gen Quant

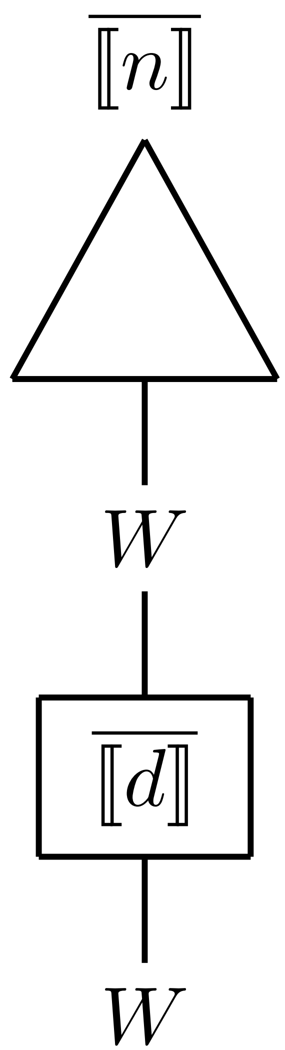
$$\overline{[\![\text{Det N}]\!]} := \overline{[\![d]\!]} \circ \overline{[\![n]\!]}$$



Compact Closed Models of Gen Quant

$$\overline{[\![\text{Det N}]\!]} := \overline{[\![d]\!]} \circ \overline{[\![n]\!]}$$

Living On



Compact Closed Models of Gen Quant

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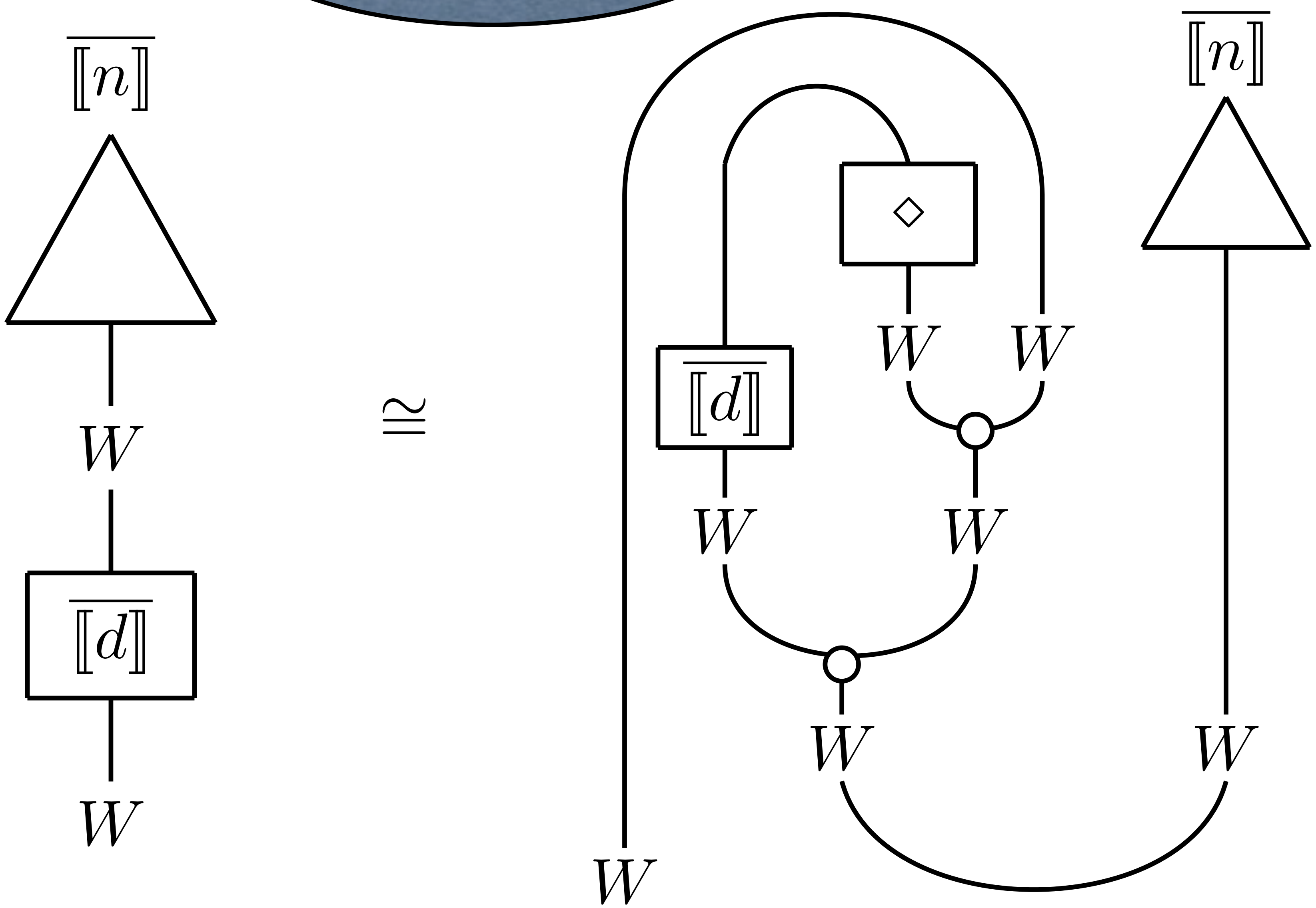
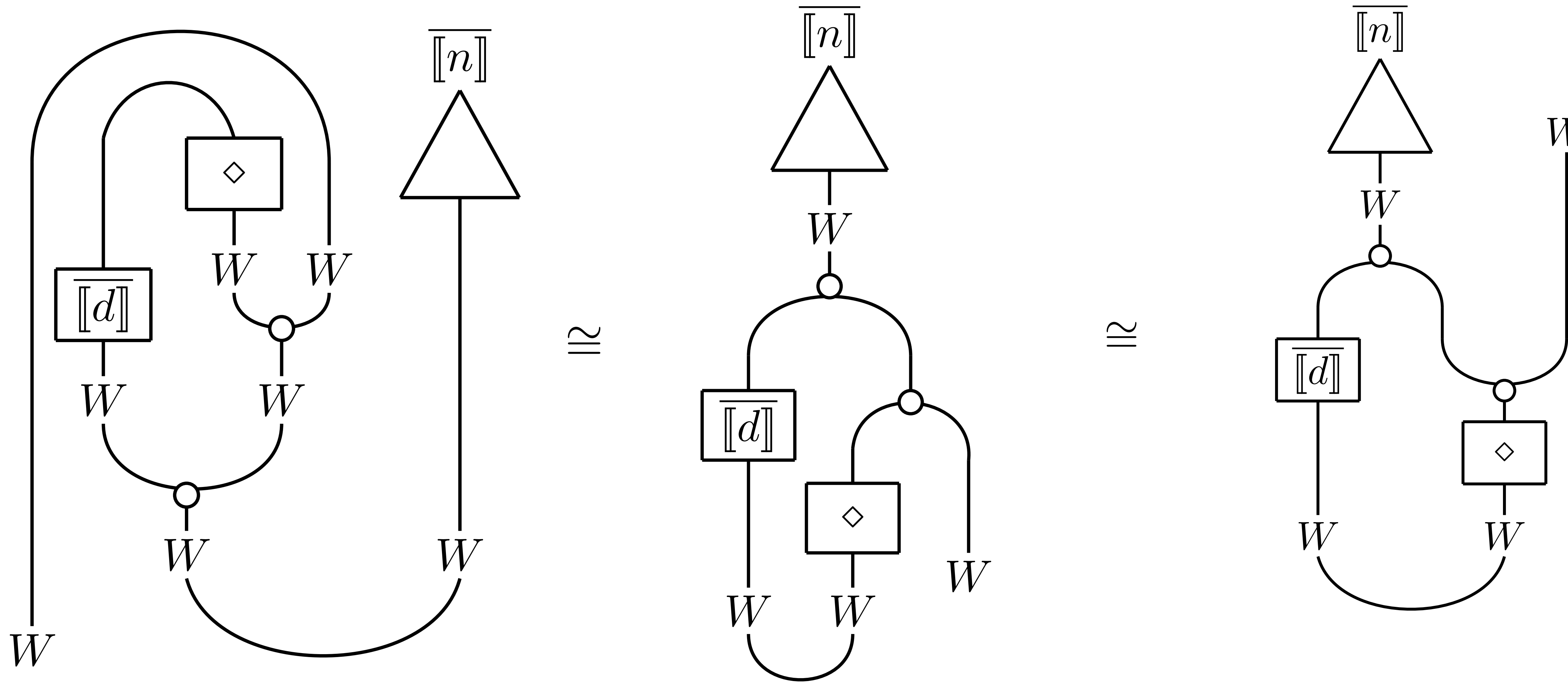
Living On



Compact Closed Models of Gen Quant

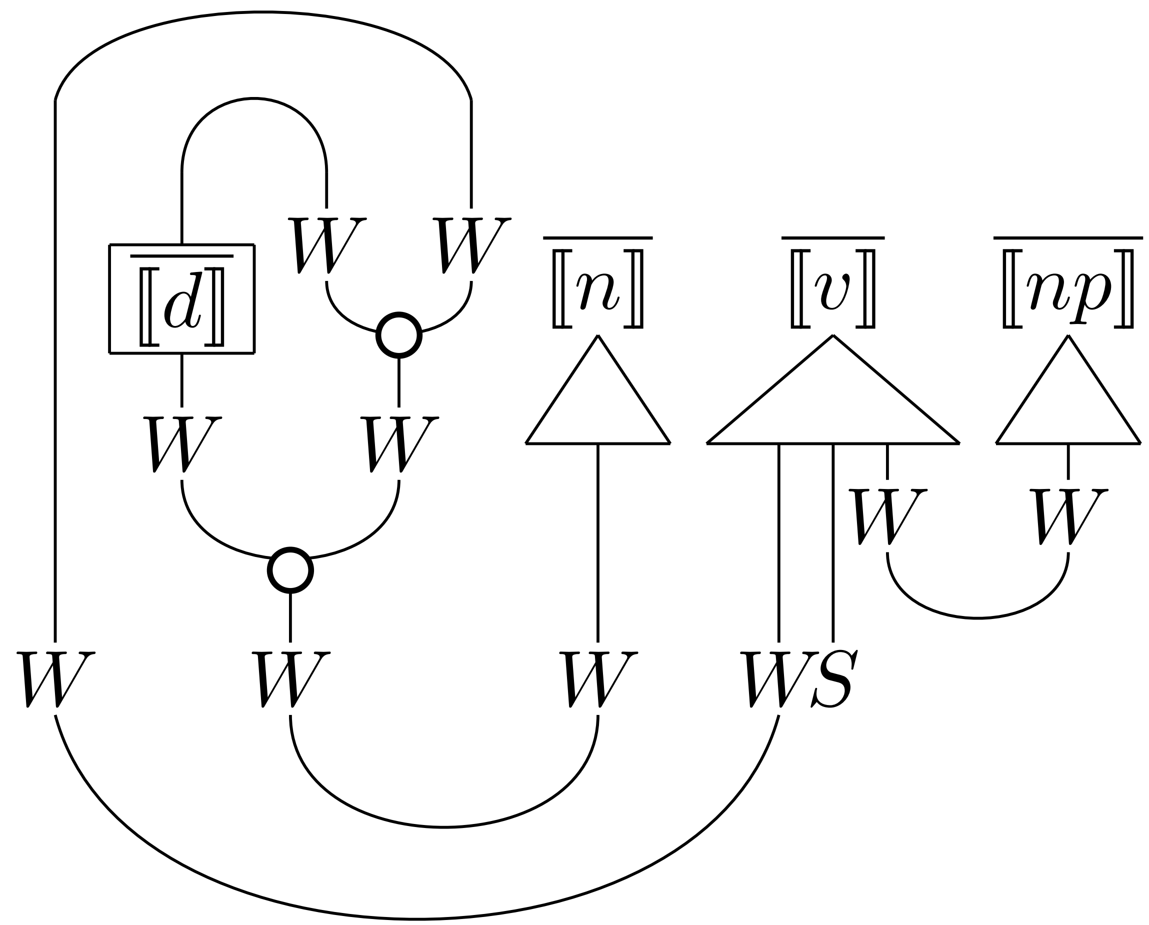
$$\overline{[\![\text{Det N}]\!]} := \overline{[\![d]\!]} \circ \overline{[\![n]\!]}$$



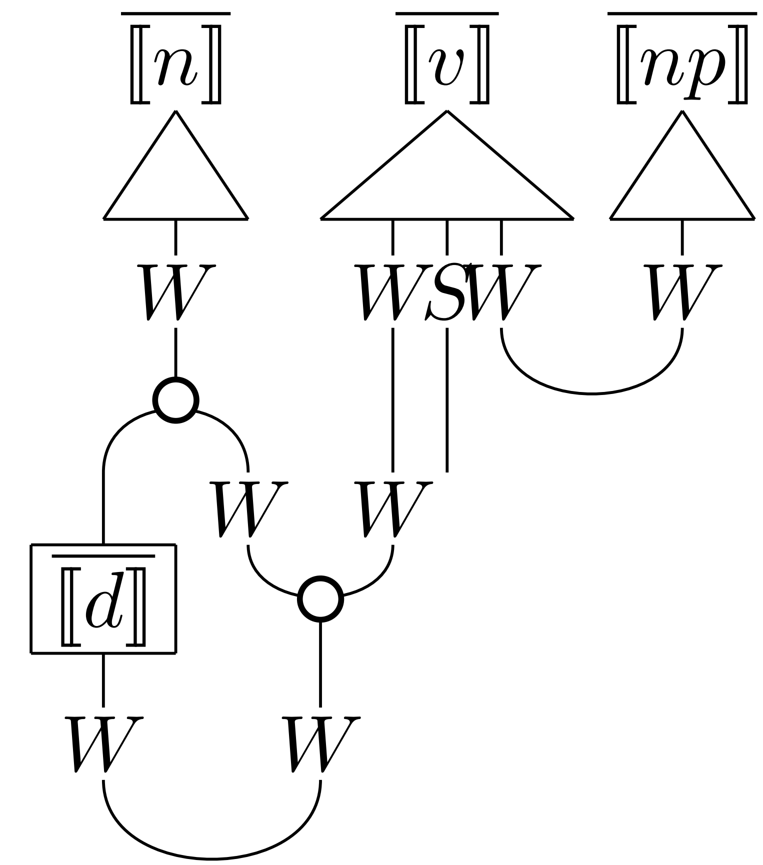
$$\cong \; \epsilon_W \circ (1_W \otimes \diamond) \circ (\overline{[\![d]\!]} \otimes \mu_W) \circ (\Delta_W \otimes 1_W) \circ (\overline{[\![n]\!]} \otimes 1_W)$$

Compact Closed Models of Gen Quant

Example: meaning of *``Det N VP NP"*



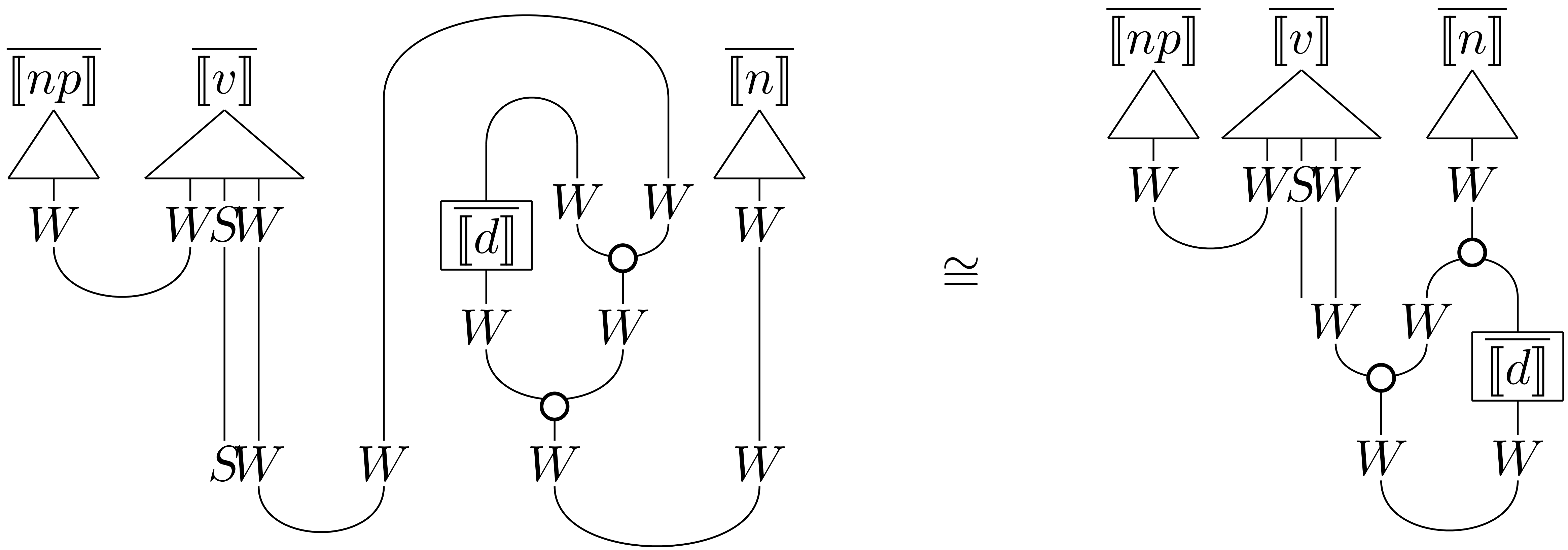
$\cong$



$$(\epsilon_W \otimes 1_S) \circ (\overline{[\![d]\!]} \otimes \mu_W \otimes 1_S) \circ (\Delta_W \otimes 1_{W \otimes S}) \circ (\overline{[\![n]\!]} \otimes \overline{[\![vp]\!]})$$

Compact Closed Models of Gen Quant

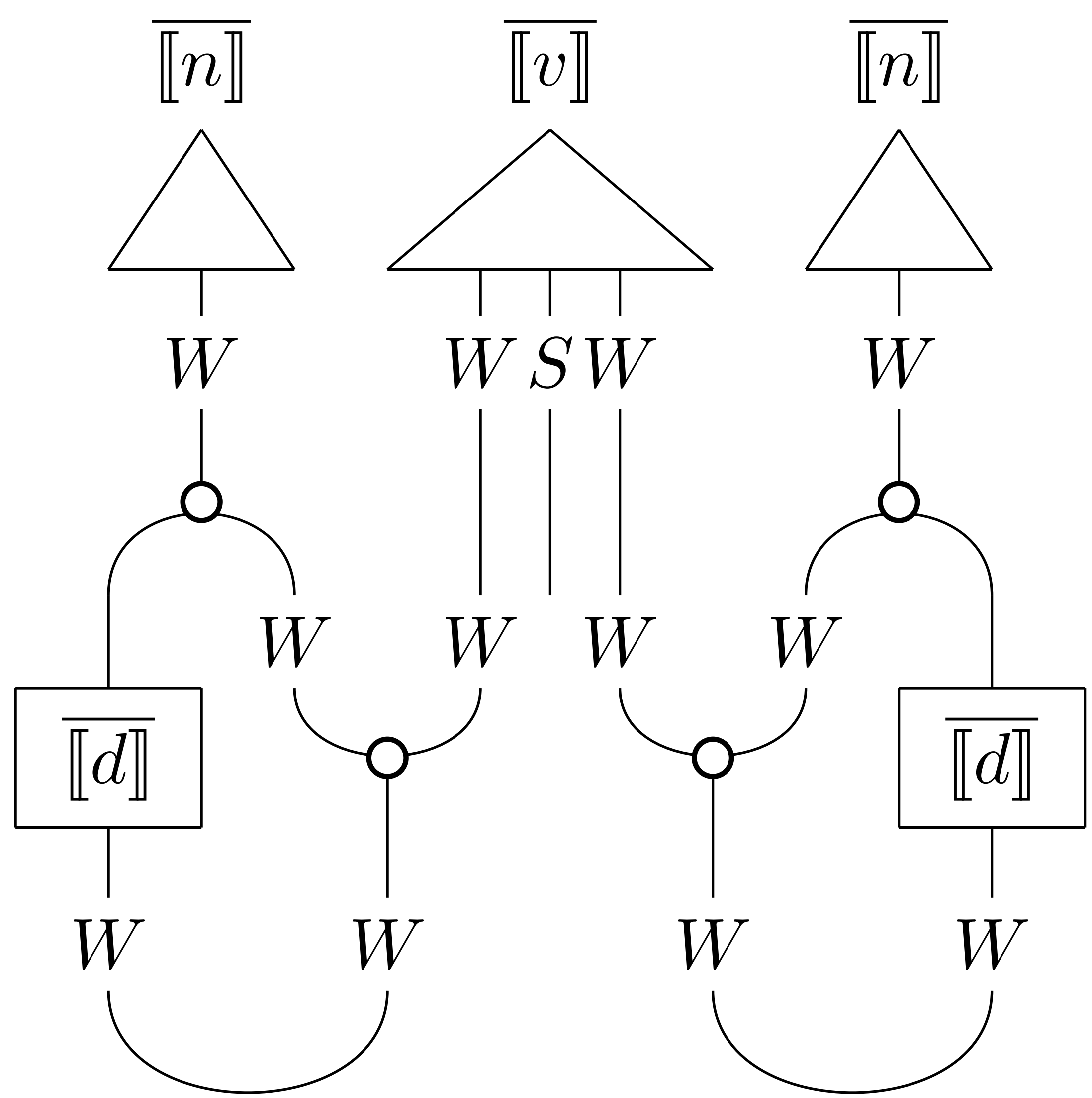
Example: meaning of *``NP V Det N''*



$$(1_S \otimes \epsilon_W) \circ (1_S \otimes \mu_W \otimes \overline{[\![d]\!]}) \circ (\epsilon_W \otimes 1_{S\otimes W} \otimes \Delta_W) \circ (\overline{[\![np]\!]} \otimes \overline{[\![v]\!]} \otimes \overline{[\![n]\!]})$$

Compact Closed Models of Gen Quant

Example: meaning of *``Det N V Det N"*



$$(\epsilon_W \otimes 1_S \otimes \epsilon_W) \circ (\overline{[\![d]\!]} \otimes \otimes \mu_W \otimes 1_S \otimes \mu_W \otimes \overline{[\![d]\!]}) \circ (\Delta_W \otimes 1_{W \otimes S \otimes W} \otimes \Delta_W) \circ (\overline{[\![n]\!]} \otimes \overline{[\![v]\!]} \otimes \overline{[\![n]\!]})$$

Truth Theoretic Model

Instantiation in sets and relations

Model $(Rel, \mathcal{P}(U), \{\star\}, \overline{[\![\ ]\!]})$

w in N, NP, VP $\overline{[\![w]\!]} : \{\star\} \to \mathcal{P}(U)$

w in V

$$\overline{[\![w]\!]} \to \mathcal{P}(U) \otimes \{\star\} \otimes \mathcal{P}(U)$$

d in Det

$$\overline{[\![d]\!]} : \mathcal{P}(U) \to \mathcal{P}(U)$$

Truth Theoretic Model
Soundness and Completeness

Definition

The compact closed meaning of a sentence in the relational interpretation is true iff $\overline{[\![NP\ VP]\!]} = \{\star\}$

Theorem

The compact closed meanings of sentences are true in the relational interpretation iff they are true in the generalized quantifier theory.

Concrete Vector Space Model

$$(\mathcal{C}, V_{\mathcal{P}(\Sigma)}, V_{\mathcal{P}(\Sigma)}, \rightarrow)$$

Main vector space $V_{\mathcal{P}(\Sigma)} = \{\overrightarrow{A}_i\}_i$

w in N, NP $\overrightarrow{\mathrm{w}} = \sum_i C_i \overrightarrow{A}_i$

w in VP $\overrightarrow{\mathrm{w}} = \Delta(\sum_{ij} C_{ij} \overrightarrow{A}_i)$

w in V $\overrightarrow{\mathrm{w}} = \Delta(\sum_{ijk} C_{ijk} \overrightarrow{A}_i \otimes \overrightarrow{A}_k)$

Definition

For d in Det we define $d(\overrightarrow{\mathrm{w}}) = \sum_o C_o \overrightarrow{A}_o$ where *Co* is the degree of statistical association between w and *d* elements of $\overrightarrow{A}_o$

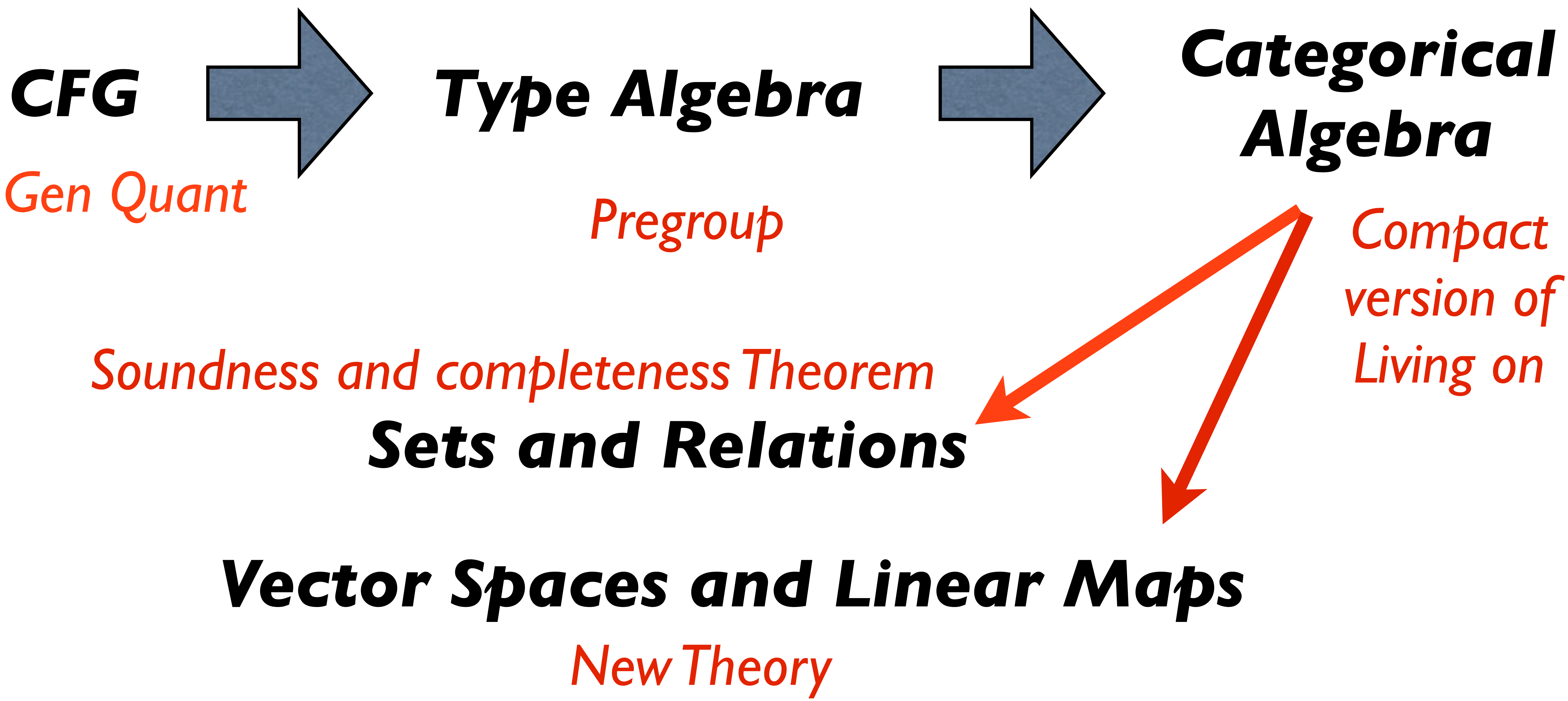
Linear Algebraic Closed Forms

$$\overrightarrow{\text{tall men}} = \overrightarrow{\text{tall}} \times \overrightarrow{\text{men}}$$

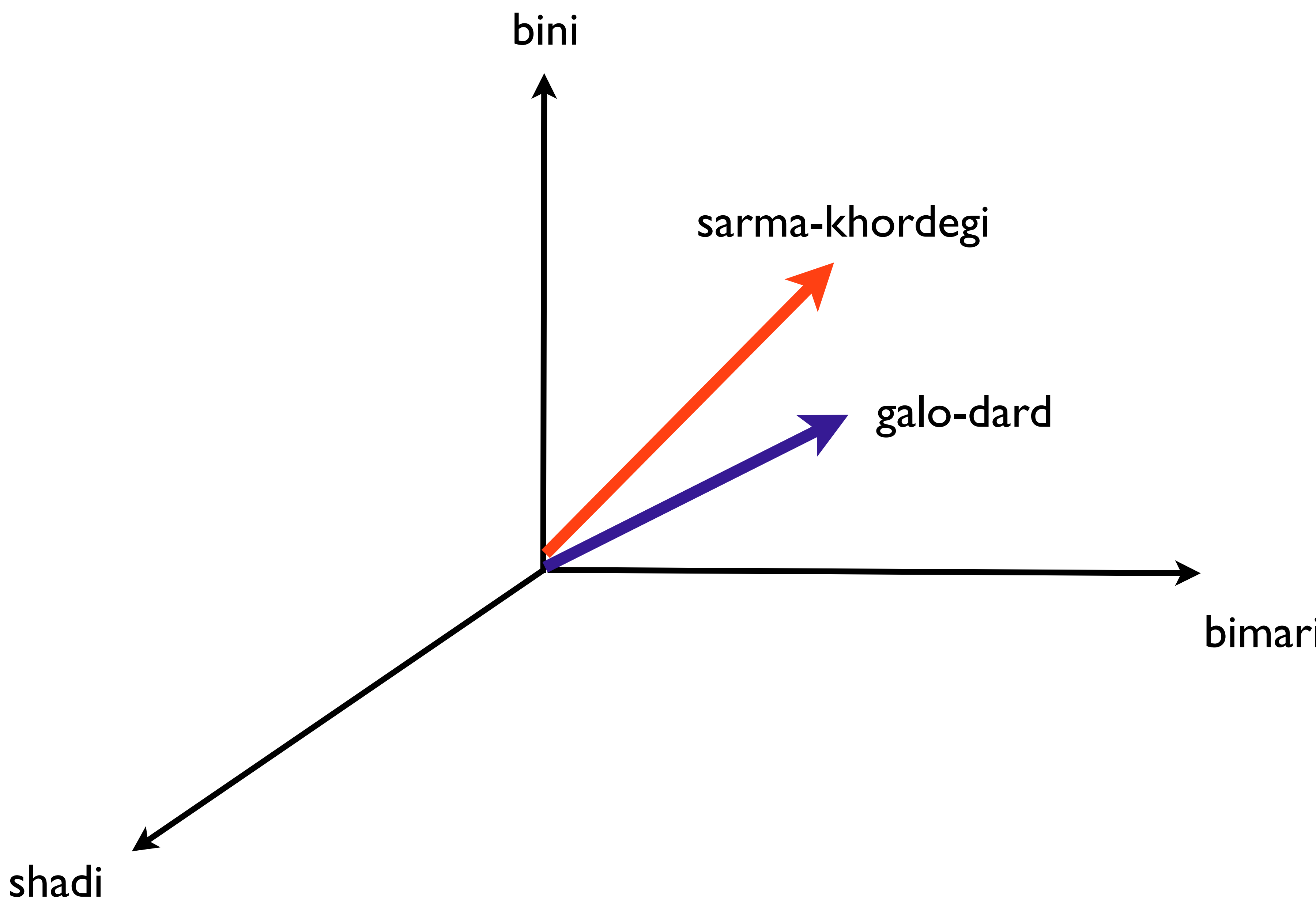
$$\overrightarrow{\text{men ate cookies}} = \overrightarrow{\text{men}}^T \times (\overrightarrow{\text{ate}} \times \overrightarrow{\text{cookies}})$$

$$\overrightarrow{\text{men who ate cookies}} = \overrightarrow{\text{men}} \odot (\overrightarrow{\text{ate}} \times \overrightarrow{\text{cookies}})$$

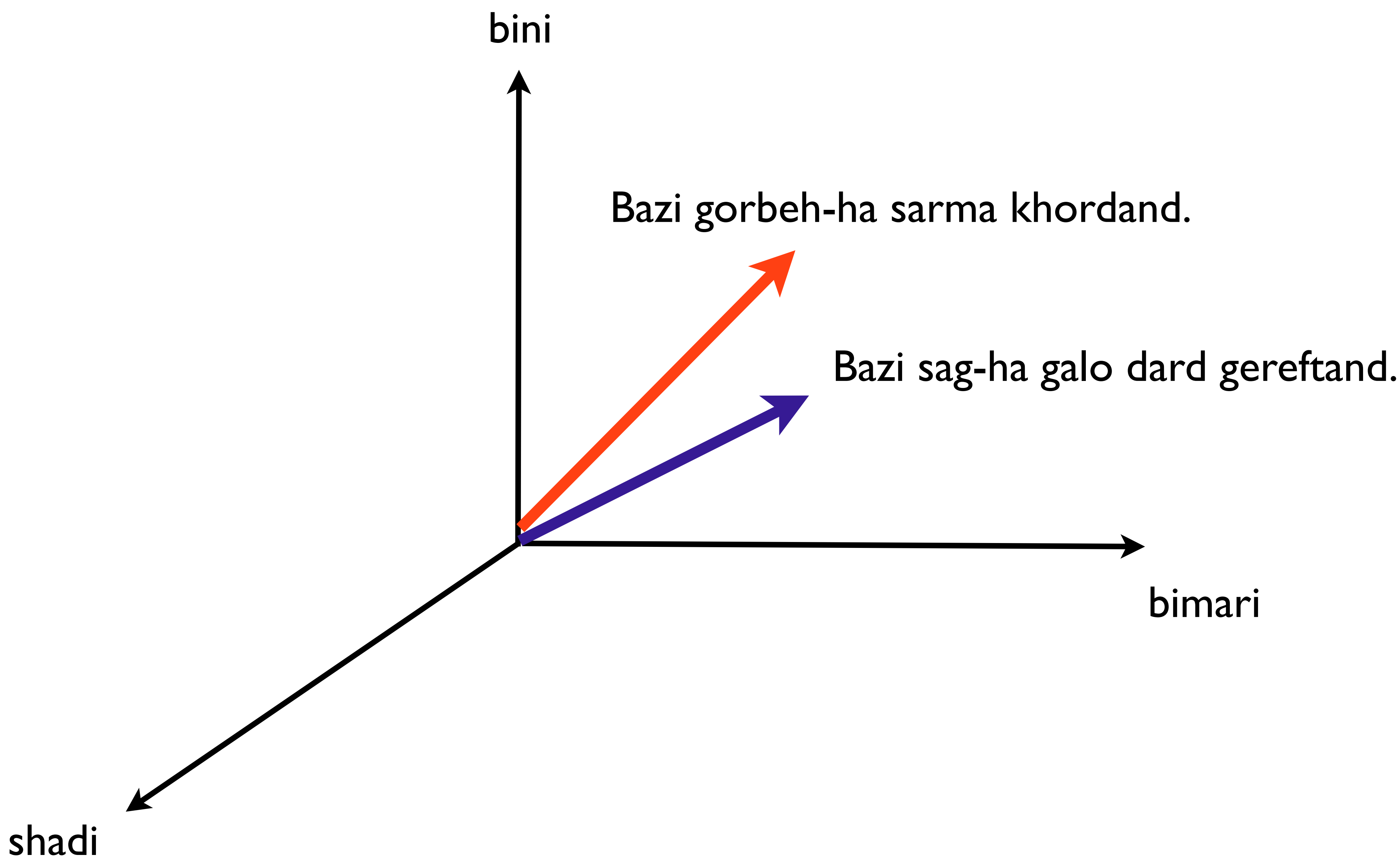
$$\overrightarrow{\text{some men ate cookies}} = \text{some}(\overrightarrow{\text{men}})^T \times (\overrightarrow{\text{ate}} \times \overrightarrow{\text{cookies}})$$


CFG
Gen Quant
Type Algebra
Pregroup
Categorical Algebra
Compact version of Living on
Soundness and completeness Theorem
Sets and Relations
Vector Spaces and Linear Maps
New Theory

Vector Representations for Words



Vector Representations for Words



Disambiguation

Grefenstette and MS, EMNLP, *Journal of Computational Linguistics*

young child draw sword	young child pulled sword
	young child sketched sword

Model	ρ
Verb Baseline	0.20
Bigram Baseline	0.14
Trigram Baseline	0.16
Additive	0.10
Multiplicative	
AdjMult	**0.20**
AdjNoun	0.05
CategoricalAdj	**0.20**
Categorical	
AdjMult	0.14
AdjNoun	0.16
CategoricalAdj	**0.19**
Kronecker	
AdjMult	**0.26**
AdjNoun	0.17
CategoricalAdj	**0.27**
Upperbound	0.48

Sentence Similarity

Kartsaklis and MS, CoNLL, EMNLP

Sentence 1	Sentence 2
man shut door	gentleman close eye
survey collect information	page provide datum
project present problem	programme face difficulty

	Model	Ambig.		Disamb.
BL	Verbs only	0.310	$\ll$	0.341
M1	Multiplicative	0.325	$\ll$	0.404
M2	Additive	0.368	$\ll$	0.410
T1	Relational	0.368	$\ll$	0.397
T2	**Kronecker**	**0.404**	$<$	**0.412**
T3	Copy-subject	0.310	$\ll$	0.337
T4	Copy-object	0.321	$\ll$	0.368
	Human agreement	0.550		

Term-Description Classification

Kartsaklis, MS, Clark, Rimmel, Coecke, MoL, JLC

	Term	Description
1	emperor	person who rule empire
2	queen	woman who reign country
3	mammal	animal which give birth
4	plug	plastic which stop water
5	carnivore	animal who eat meat
6	vegetarian	person who prefer vegetable
7	doll	toy that girl prefer
8	football	game that boy like
9	skirt	garment that woman wear
10	widow	woman whose husband die
11	orphan	child whose parent die
12	teacher	person whose job educate children
13	comedian	artist whose joke entertain people
14	priest	clergy whose sermon people follow
15	commander	military whose order marine obey
16	clown	entertainer whose trick people enjoy

Results

Term-Description Classification

	Base	Frob $\overline{\text{'s}} = Id$	Frob $\overline{\text{s}} = \sum_i (\overrightarrow{\text{noun}_1})_i$	Mult without Rel. Pr.	Mult with Rel. Pr.	Add /with without Rel. Pr.
MRR	0.70	0.82	0.71	0.78	0.76	0.75
Acc	0.56	0.75	0.56	0.62	0.62	0.62

Wrong Classifications?

Term	Description	Cosine
football	game that boys like	0.62
	woman who reigns country	0.24
	toy that girls prefer	0.18
	woman whose husband died	0.18
priest	clergy whose sermon people follow	0.53
	woman who reigns country	0.45
	woman whose husband died	0.37
	person who rules empire	0.35
plug	toy that girls prefer	0.24
	plastic which stops water	0.22
	woman who reigns country	0.17
	game that boys like	0.17
clown	woman who reigns country	0.28
	toy that girls prefer	0.24
	woman whose husband died	0.24
	game that boys like	0.24

Entailment

Definition *(Entailment via Feature Inclusion)*

$\overrightarrow{v} \vdash \overrightarrow{w}$ whenever features of *v* are included in features of *w*.

Proposition

If $\overrightarrow{v} \vdash \overrightarrow{w}$ and $d \vdash d'$ then it follows that $d(\overrightarrow{v}) \vdash d'(\overrightarrow{w})$

for a feature defined to be a non-zero context.

Empirical Evaluation

Pairs of entailed words harvested from corpus

government $\implies$ body	bill $\implies$ program	war $\implies$ conflict
mortgage $\implies$ loan	town $\implies$ location	

Pairs of entailed quantifiers

all $\vdash$ some much $\vdash$ some	all $\vdash$ several many $\vdash$ some	all $\vdash$ many several $\vdash$ some

some $\nvdash$ all some $\nvdash$ many	several $\nvdash$ all few $\nvdash$ many	few $\nvdash$ all

Empirical Evaluation

Experiment with entailed noun phrases

all bills $\vdash$ some programs	all bills $\vdash$ several programs	all wars $\vdash$ many conflicts
some bills $\nvdash$ all programs	several bills $\nvdash$ all programs	few wars $\nvdash$ all conflicts

Experiment with entailed sentences

all bills fail. $\vdash$ some programs fail.	all wars are bad. $\vdash$ many conflicts are bad.
some bills fail. $\nvdash$ all programs fail.	few wars are worth fighting. $\nvdash$ all conflicts are worth fighting.

Conclusions

Abstract categorical model for NL

Instantiations

sets and relations: set-theory

vector spaces: pragmatics

Extending the pragmatic model from words to sentences

some Empirical Evaluation

Mersi!

Collaborators and Papers

Dmitrijs Milajevs, Dimitri Kartsaklis, Mehrnoosh Sadrzadeh, Matthew Purver, 'Evaluating Neural Word Representations in Tensor-Based Compositional Settings', Conference on Empirical Methods in Natural Language Processing (EMNLP), Qatar, October 2014.

D. Kartsaklis, N. Kalchbrenner and M. Sadrzadeh, 'Resolving Lexical Ambiguity in Tensor Regression Models of Meaning', In Proceedings of the 52nd Annual Meeting of the Association for Computational Linguistics (ACL), Baltimore USA, June, 2014.

D. Kartaklis and M. Sadrzadeh, 'Prior Disambiguation of Word Tensors for Constructing Sentence Vectors', Conference on Empirical Methods in Natural Language Processing (EMNLP), Seattle, October 2013.

E. Grefenstette and M. Sadrzadeh, 'Experimental Support for a Categorical Compositional Distributional Model of Meaning', Conference on Empirical Methods in Natural Language Processing (EMNLP), pp. 1394-1404, 2011, Edinburgh.

Collaborators and Papers

E. Grefenstette and M. Sadrzadeh, 'Concrete Models and Empirical Evaluations for the Categorical Compositional Distributional Model of Meaning', Journal of Computational Linguistics, to appear.

M. Sadrzadeh, S. Clark, B. Coecke, 'Frobenius Anatomy of Word Meaning II: possessive relative pronouns', *Journal of Logic and Computation*, Special Issue dedicated to Roy Dyckhoff, to appear.

M. Sadrzadeh, S. Clark, B. Coecke, 'Frobenius Anatomy of Word Meaning I: subject and object relative pronouns', *Journal of Logic and Computation*, Special Issue: The Incomputable, an Isaac Newton Institute Workshop, **23**, pp. 1293-1317, 2013.

B. Coecke, E. Grefenstette and M. Sadrzadeh, 'Lambek vs. Lambek: Vector Space Semantics and String Diagrams for Lambek Calculus', *Annals of Pure and Applied Logic*, Volume 164, Issue 11, November 2013, Pages 1079–1100.


Google
dog
Search
About 1,400,000,000 results (0.13 seconds)
Advanced search
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Dog - Wikipedia, the free encyclopedia
The domestic dog (Canis lupus familiaris and Canis lupus dingo) is a domesticated form of the gray wolf, a member of the Canidae family of the order ...
List of dog breeds - Origin of the domestic dog - List of most popular dog breeds
en.wikipedia.org/wiki/Dog - Cached - Similar -
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Police say Spotsylvania **man killed** by **his dog** | Richmond Times **...**
15 Jun 2011 **...** Police say a Spotsylvania County **man** found dead in **his** home was **killed** by **his** pit bull.
www2.timesdispatch.com/.../police-say-spotsylvania-**man-killed-his-dog**-ar-1109796/ - Cached - Block all www2.timesdispatch.com results

Translate CFG to Type Algebra

CFG → **PRG**

$$G = (T, N, S, \mathcal{R})$$ → $$P = (P, \Sigma, \beta, s)$$

$$\sigma : T \cup N \to P$$

l-t-r $A \to BC$ → $\sigma(C) = \sigma(B)^r \cdot \sigma(A)$

r-t-l $A \to BC$ → $\sigma(B) = \sigma(A) \cdot \sigma(C)^l$

Terminals $A \to a$ → $\sigma(a) = \sigma(A)$

End Sym → $\sigma(S) = s$

Translate CFG to Type Algebra

CFG → **PRG**

$S \to NP\ VP$ ⇒ $\sigma(VP) = \sigma(NP)^r \cdot \sigma(S)$

$VP \to V\ NP$ ⇒ $\sigma(V) = \sigma(VP) \cdot \sigma(NP)^l$

$NP \to Det\ N$ ⇒ $\sigma(Det) = \sigma(NP) \cdot \sigma(N)^l$

Terminals ⇒ $\sigma(NP) = p \qquad \sigma(N) = m$

End Symbol ⇒ $\sigma(S) = s$

Truth Theoretic Model

Instantiation in sets and relations

Model $(Rel, \mathcal{P}(U), \{\star\}, \overline{[\![\ ]\!]})$

w in N, NP, VP

$$\overline{[\![w]\!]} \colon \{\star\} \to \mathcal{P}(U)$$

$$\overline{[\![w]\!]} := \{\{a\} \mid \text{for each } a \in [\![w]\!]\}$$

w in V

$$\overline{[\![w]\!]} \to \mathcal{P}(U) \otimes \{\star\} \otimes \mathcal{P}(U)$$

$$\overline{[\![w]\!]} := \{(\{a\}, \{b\}) \mid (a, b) \in [\![w]\!]\}$$

d in Det

$$\overline{[\![d]\!]} \colon \mathcal{P}(U) \to \mathcal{P}(U)$$

the relation corresponding to $[\![d]\!]$

Truth Theoretic Model

Example

Meaning of ``Det N VP'' is

$$\epsilon_{\mathcal{P}(U)} \circ (1_{\mathcal{P}(U)} \otimes \diamond) \circ (\overline{[\![d]\!]} \otimes \mu_{\mathcal{P}(U)}) \circ (\Delta_{\mathcal{P}(U)} \otimes 1_{\mathcal{P}(U)} \otimes \epsilon_{\mathcal{P}(U)})$$

Suppose we have 2 individuals m_1, m_2 and one cat c_1

$$\overline{[\![\text{men}]\!]} = \{\{m_1\}, \{m_2\}\} \qquad \overline{[\![\text{cat}]\!]} = \{\{c_1\}\}$$

The cat and the first man sneeze. $\overline{[\![\text{sneeze}]\!]} = \{\{m_1\}, \{c_1\}\}$

Some and all:

$$Some\Big([\![\text{men}]\!]\Big) = \{\{m_1\}, \{m_2\}, \{m_1, m_2\}\}$$

$$All\Big([\![\text{man}]\!]\Big) = \{\{m_1, m_2\}\}$$

Truth Theoretic Model

Example

Meaning of \`some men sneeze' is computed as follows:

1- $(\Delta \otimes 1)\Big(\overline{[\![\text{men}]\!]} \otimes \overline{[\![\text{sneeze}]\!]}\Big) =$

$$\{(\{m_1\},\{m_1\}),(\{m_2\},\{m_2\})\} \otimes \{\{m_1\},\{c_1\}\}$$

2- apply $\Big(\overline{[\![Some]\!]} \otimes \mu\Big)$ to the above

$$\{\{m_1\},\{m_2\},\{m_1,m_2\}\} \otimes \{\{m_1\}\}$$

3- apply ϵ to the above

$$\epsilon\Big(\{\{m_1\},\{m_2\},\{m_1,m_2\}\} \otimes \{\{m_1\}\}\Big) = \{\star\}$$