درستی جملات گودلی و راسِری

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(کاری مشترک با کاوه‌لاجوردی و زیبااسعدی‌گلزار)

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سخنرانی (مَجازی) ماهانه‌ی انجمن منطق ایران

قضیه (اول) ناتمامیت گودل

Gödel, Kurt (1931); *Über Formal Unentscheidbare Sätze der* Principia Mathematica *und Verwandter Systeme I*, **Monatshefte für Mathematik und Physik** 38(1):173–198 (in German). II?
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- intended to publish it in the *Bulletin of the AMS*, never happened!

“ So the following disjunctive conclusion is inevitable: *Either mathematics is incompletable in this sense, that its evident axioms can never be comprised in a finite rule, that is to say, the human mind (even within the realm of pure mathematics) infinitely surpasses the powers of any finite machine, or else there exist absolutely unsolvable diophantine problems of the type specified* (where the case that both terms of the disjunction are true is not excluded, so that there are, strictly speaking, three alternatives). ”

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یادش به خیر ...

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On the Arithmetical Truth of Self-Referential Sentences

by

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Abstract: We take an argument of Gödel's from his ground-breaking 1931 paper, generalize it, and examine its validity. The argument in question is this: *the sentence G says about itself that it is not provable, and G is indeed not provable; therefore, G is true.*

Keywords: Gödel's first incompleteness theorem, the Gödel sentence, self-reference, truth, arithmetic, soundness, ω-consistency

جمله گودل نه ... (بلکه) جملات گودلی

How Not to Define *the* Gödel Sentence of a Theory: A Very Short Note

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Abstract. We argue that, under the usual assumptions for theories subject to Gödel's First Incompleteness Theorem, one cannot, without impropriety, talk about *the* Gödel sentence of the theory, for there could be a true sentence and a false one, each of which equivalent to its own unprovability in the theory.

§**1. Introduction.** In the course of proving what is now known as the *First Incompleteness Theorem* (Gödel 1931) [Theorem VI, page 173] for any theory T satisfying certain conditions, Gödel constructs a sentence which is, in the eye of T, equivalent to its own unprovability—that is to say, a sentence G_T such that

$$T \vdash \mathsf{G}_T \leftrightarrow \neg \mathtt{Pr}_T(\ulcorner \mathsf{G}_T \urcorner),$$

where $\mathtt{Pr}_T(x)$ is the provability predicate of T, and $\ulcorner A \urcorner$ denotes the numeral corresponding to the Gödel number of A. We assume that this is all done relative to a fixed Gödel numbering and a fixed Σ_1-arithmetization of T, thus making $\mathtt{Pr}_T(x)$ a Σ_1-formula.

چونکه هر نظریه ناصحیح جملات درست و نادرست گودلی دارد

Theorem 5 *All the Gödelian sentences of a theory T are true if and only if T is sound.*

Proof. If the thoery T is sound and φ is a Gödelian sentence of T, then $\mathbb{N} \vDash \varphi \leftrightarrow \neg \mathrm{Pr}_T(\ulcorner \varphi \urcorner)$. On the other hand, from $T \nvdash \varphi$ (Theorem 4) it follows that $\mathbb{N} \vDash \neg \mathrm{Pr}_T(\ulcorner \varphi \urcorner)$. Thus, $\mathbb{N} \vDash \varphi$.

Now, suppose that all the Gödelian sentences of the theory T are true. The theory T is consistent, since otherwise every (false) sentence would be a Gödelian sentence of it. We show that T is sound. Suppose that $T \vdash \theta$ for a sentence θ; we aim at showing the truth of θ, i.e. that $\mathbb{N} \vDash \theta$. By the Diagonal Lemma there exists a sentence η such that

$$Q \vdash \eta \leftrightarrow [\theta \leftrightarrow \neg \mathrm{Pr}_T(\ulcorner \eta \urcorner)].$$

Hence $T \vdash \eta \leftrightarrow \neg \mathrm{Pr}_T(\ulcorner \eta \urcorner)$, and so η is a Gödelian sentence of T. Thus $\mathbb{N} \vDash \eta$ by the assumption. As a result (of the soundness of Q) we have $\mathbb{N} \vDash \theta \leftrightarrow \neg \mathrm{Pr}_T(\ulcorner \eta \urcorner)$. On the other hand, by Theorem 4, we have $T \nvdash \eta$ and so $\mathbb{N} \vDash \neg \mathrm{Pr}_T(\ulcorner \eta \urcorner)$. Therefore, $\mathbb{N} \vDash \theta$. □

So, if a theory is unsound, then it must have some false Gödelian sentences. Below we show that it must have some true Gödelian sentences as well.

Corollary 6 *Every unsound theory has at least one true and one false Gödelian sentences.*

Proof. If U is inconsistent, then every sentence is a Gödelian sentence of U. So, suppose that the unsound theory U is consistent; there exists a false Gödelian sentence for U by Theorem 5. By the Diagonal Lemma, for some sentence γ we have $Q \vdash \gamma \leftrightarrow \neg \mathrm{Pr}_U(\ulcorner \gamma \urcorner)$. Now, γ is a Gödelian sentence of U, and so by Theorem 4 we have $\mathbb{N} \vDash \neg \mathrm{Pr}_U(\ulcorner \gamma \urcorner)$. Thus $\mathbb{N} \vDash \gamma$ (by the soundness of Q), and so γ is a true Gödelian sentence of U. □

بالاخره: درستی جملات گودلی (و راسِری)

On the Truth of Gödel Sentences and Rosser Sentences

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Abstract. There is a longstanding debate in the logico-philosophical community as to why the Gödel sentences of a consistent and sufficiently strong theory are true. The prevalent argument seems to be something like this: since every one of the Gödel sentences of such a theory is equivalent to the theory's consistency statement, even provably so inside the theory, the truth of those sentences follows from the consistency of the theory in question. So, Gödel sentences of consistent theories should be true. In this paper, we show that Gödel sentences of only sound theories are true; and there is a long road from consistency to soundness, indeed a hierarchy of conditions which are satisfied by some and are falsified by others. We also study the truth of Rosser sentences and provide necessary and sufficient conditions for the truth of Rosser (and also Gödel) sentences of arithmetical theories.

Keywords: The Incompleteness Theorem; Gödel Sentences, Rosser's Trick, Rosser Sentences, Soundness, Consistency, Σ_n-Soundness.
2020 Math Subject Classification: 03F40.

در باب اهمیت جملات گودلی

Proposition 5 (Characterizing Unprovable Sentences) *Suppose that Löb's Rule holds for the theory T. The following are equivalent for every sentence φ:*

(1) φ is unprovable in T, i.e., $T \nvdash \varphi$;
(2) φ is a Gödel sentence of some consistent extension U of T;
(3) $T + [\varphi \leftrightarrow \neg \mathtt{Pr}_T(\#\varphi)]$ is consistent.

Proof.
$(1 \Rightarrow 2)$: There exists, by Lemma 1, a sentence ξ such that $T \vdash \xi \leftrightarrow [\varphi \leftrightarrow \neg \mathtt{Pr}_{T+\xi}(\#\varphi)]$. Let $U = T + \xi$; then $U \vdash \varphi \leftrightarrow \neg \mathtt{Pr}_U(\#\varphi)$ and it remains to show that U is consistent. If not, then $T \vdash \neg\xi$. So, on the one hand we have (i) $T \vdash \neg[\varphi \leftrightarrow \neg \mathtt{Pr}_U(\#\varphi)]$, and on the other hand $U \vdash \varphi$ which implies (ii) $T \vdash \mathtt{Pr}_U(\#\varphi)$ by Convention 2. Now, (i) and (ii) imply that $T \vdash \varphi$, contradicting the assumption.
$(2 \Rightarrow 3)$: If $T + [\varphi \leftrightarrow \neg \mathtt{Pr}_T(\#\varphi)]$ is not consistent, then $T \vdash \neg[\varphi \leftrightarrow \neg \mathtt{Pr}_T(\#\varphi)]$, and so $T \vdash \mathtt{Pr}_T(\#\varphi) \rightarrow \varphi$, which implies $T \vdash \varphi$ by Löb's Rule. So, for every extension U of T we have $U \vdash \varphi$, and so, by Convention 2, $U \vdash \mathtt{Pr}_U(\#\varphi)$. Therefore, for every such U we have $U \vdash \neg[\varphi \leftrightarrow \neg \mathtt{Pr}_U(\#\varphi)]$, which contradicts the assumption.
$(3 \Rightarrow 1)$: If $T \vdash \varphi$, then, by Convention 2, we have $T \vdash \mathtt{Pr}_T(\#\varphi)$, and so we should have also $T \vdash \neg[\varphi \leftrightarrow \neg \mathtt{Pr}_T(\#\varphi)]$. □

It should be noted that the assumption of holding Löb's Rule for T was used only in the implication $(2 \Rightarrow 3)$. So, (1) and (2) are equivalent with each other, and are implied by (3), even when this rule does not hold; cf. Theorem 4 of (Lajevardi & Salehi 2020).

تظریف درستی جملات گودلی

Lemma 9 (On Extensions of Υ-Sound Theories) *Let Υ be a class of sentences that is closed under disjunction. If T is a Υ-sound theory, then for every sentence φ, either $T+\varphi$ or $T+\neg\varphi$ is Υ-sound.*

Proof. If neither $T+\varphi$ nor $T+\neg\varphi$ is Υ-sound, then for some false Υ-sentences ς and ς' we have $T+\varphi\vdash\varsigma$ and $T+\neg\varphi\vdash\varsigma'$. Thus, $T\vdash\varsigma\vee\varsigma'$, and $\varsigma\vee\varsigma'$ is a false Υ-sentence; a contradiction with the Υ-soundness of T. □

Our main results is the following necessary and sufficient condition for the truth of Gödel (Π_{n+1} and Σ_{n+1}) sentences:

Theorem 10 (The Truth of Gödel Sentences) *Let $n\geqslant 1$.*
All the Gödel Π_{n+1}-sentences of T are true if and only if T is Π_{n+1}-sound.
All the Gödel Σ_{n+1}-sentences of T are true if and only if T is Σ_{n+1}-sound.

Proof. Let Υ be any of Π_{n+1} or Σ_{n+1}.

First, suppose that T is Υ-sound, and let γ be a Gödel Υ-sentence of T. By Lemma 4 and Convention 2 we have $\mathbb{N}\vDash\neg\mathtt{Pr}_T(\#\gamma)$, and so $\mathtt{Pr}_T(\#\gamma)$ is a false Σ_1-sentence. Now, $T+\neg\gamma\vdash\mathtt{Pr}_T(\#\gamma)$, and so $T+\neg\gamma$ is not Σ_1-sound; whence, it is not Υ-sound either. Thus, by Lemma 9, the theory $T+\gamma$ should be Υ-sound. Therefore, γ must be true.

Now, suppose that all the Gödel Υ-sentences of T are true. We show that the theory T is Υ-sound. Assume that $T\vdash\varsigma$ for a Υ-sentence ς. We prove that ς is true. By Lemma 1 there exists a Υ-sentence ζ such that $\boldsymbol{Q}\vdash\zeta\leftrightarrow[\varsigma\wedge\neg\mathtt{Pr}_T(\#\zeta)]$. Thus, from $T\vdash\varsigma$ we have $T\vdash\zeta\leftrightarrow\neg\mathtt{Pr}_T(\#\zeta)$, and so ζ is a Gödel Υ-sentence of T. Whence, ζ is true, and so, by the soundness of $\boldsymbol{Q}$, we have $\mathbb{N}\vDash\varsigma$. □

برهانی دگر بر درستی جملات گودلی فقط در نظریه‌های صحیح (لم)

Lemma. If τ is a T-provable sentence and γ is a Gödel sentence of T, then $\tau\wedge\gamma$ is a Gödel sentence of T as well.

Proof. By the T-provability of τ we have $T\vdash(\tau\wedge\gamma)\leftrightarrow\gamma$. Therefore, by the one the well-known derivability conditions, $T\vdash \mathrm{Pr}_T(\#[\tau\wedge\gamma])\leftrightarrow\mathrm{Pr}_T(\#\gamma)$ hence $T\vdash\neg\mathrm{Pr}_T(\#[\tau\wedge\gamma])\leftrightarrow\neg\mathrm{Pr}_T(\#\gamma)$. Since γ is a Gödel sentence of T, we have

$$T\vdash(\tau\wedge\gamma)\leftrightarrow\gamma\leftrightarrow\neg\mathrm{Pr}_T(\#\gamma)\leftrightarrow\neg\mathrm{Pr}_T(\#[\tau\wedge\gamma]),$$

which shows that $\tau\wedge\gamma$ is a Gödel sentence of T.

QED.

برهانی دگر بر درستی جملات گودلی فقط در نظریه‌های صحیح

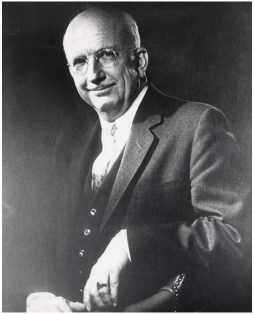
Theorem. All the Gödel sentence of T are true if and only if T is sound.

Proof. If γ is a Gödel sentence of a sound theory T, then $T \vdash \gamma \leftrightarrow \neg \mathrm{Pr}_T(\#\gamma)$ implies that the biconditional $\gamma \leftrightarrow \neg \mathrm{Pr}_T(\#\gamma)$ is a true sentence. By the T-unprovability of γ (Gödel's proof), the sentence $\neg \mathrm{Pr}_T(\#\gamma)$ is true. Therefore γ is true.

On the other hand, if T is not sound then there is a false T-provable sentence τ. Let γ be any Gödel sentence of T (whose existence is shown by the diagonal lemma). Then $\tau \wedge \gamma$ is a false sentence which is, by the Lemma, a Gödel sentence of T. Therefore T has a false Gödel sentence.

QED.

تقویت قضیه ناتمامیت گودل توسط راسِر



Rosser, J. Barkley (1936); *Extensions of Some Theorems of Gödel and Church*, **The Journal of Symbolic Logic** 1(3):87–91.

جمله راسِری: خواص و درستی

Proposition 17 (Characterizing Independent Sentences) *Let φ be a sentence. The following are equivalent:*

(1) φ is independent from T, i.e., $T \nvdash \varphi$ and $T \nvdash \neg\varphi$;
(2) φ is a Rosser sentence of some consistent extension U of T;

Theorem 19 (On the Truth of the Rosser Π_1, Σ_1-Sentences) *Every Rosser Π_1-sentence of T is true, and every Rosser Σ_1-sentence of T is false, if T is consistent.*

Theorem 20 (On the Truth of the Rosser Π_{n+1}, Σ_{n+1}-Sentences) *Let $n \geqslant 1$ be fixed. All the Rosser Π_{n+1}-sentences of T are true if and only if T is Π_{n+1}-sound. All the Rosser Σ_{n+1}-sentences of T are true if and only if T is Σ_{n+1}-sound.*

جمع‌بندی:

	Soundness	$\equiv$ Truth of all the Gödel and Rosser Sentences
$\vdots$	$\vdots$	$\vdots$
Σ_{n+1}–Soundness $\equiv$	Π_{n+2}–Soundness	$\equiv$ Truth of Gödel & Rosser Σ_{n+1} & Π_{n+2} Sentences
$\vdots$	$\vdots$	$\vdots$
Σ_2–Soundness $\equiv$	Π_3–Soundness	$\equiv$ Truth of Gödel & Rosser Σ_2 & Π_3 Sentences
Σ_1–Soundness $\equiv$	Π_2–Soundness	$\equiv$ Truth of Gödel & Rosser Π_2 Sentences
Consistency of $T+\texttt{Con}_T$		$\equiv$ Truth of Gödel Π_1 Sentences
Consistency $\equiv$	Π_1–Soundness	$\equiv$ Truth of Rosser Π_1 Sentences
Consistency $\equiv$	Π_1–Soundness	$\equiv$ Falsity of Gödel & Rosser Σ_1 Sentences

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سپاس

با سپاس فراوان

از شرکت‌کنندگان عزیز

و برگزارکنندگان گرامی

پذیرایی مختصر (چند ازین الفاظ و اضمار و مجاز / سوز خواهم سوز با آن سوز ساز)