غفلت از هندسه در فلسفهٔ ریاضی معاصر

سیاوش شهشهانی

۲۶ مهر ۱۴۰۰

انجمن منطق ایران

Aristotle: Posterior Analytics, I.7, 40 (trans. G.R.G.Mure)

It follows that we cannot in demonstrating pass from one genus to another. We cannot, for instance, prove geometrical truths by arithmetic. For there are three elements in demonstration: (1) what is proved, the conclusion – an attribute inhering essentially in a genus; (2) the axioms, i.e., the axioms which are premises of demonstration; (3) the subject genus whose attributes, i.e., essential properties, are revealed by the demonstration. The axioms which are the premises of demonstration may be identical in two or more sciences: but in the case of two different genera such as arithmetic and geometry you cannot apply arithmetical demonstration to the properties of magnitudes unless the magnitudes in question are numbers.

خوارزمی، خیام و دکارت

Here I beg you to observe in passing that the scruples that ancient writers observed in using arithmetical terms in geometry, thus making it impossible for them to proceed beyond a point where they could see clearly the relation between the two subjects, caused much obscurity and embarrassment, in their attempts at explanation.

I would borrow the best of geometry and of algebra and correct all the faults of the one by the other.

- R. Descartes: ***Geometry***, 1637.

B. Bolzano: *Considerations on the objects of elementary geometry*, 1804

I stipulate the rule that the obviousness of a proposition does not absolve me from the obligation still to look for a proof of it, …

one must regard the endeavor of unfolding all truths of mathematics down to their ultimate grounds, and thereby providing all concepts of this science with the greatest possible clarity, correctness, and order …

if the first ideas are clearly and correctly grasped then much more can be deduced from them than if they remain confused …

R. Dedekind: *Continuity and irrational numbers,* 1872

… I made the fixed resolve to keep meditating on the question [appeal to geometric intuition] till I should find a purely arithmetic and perfectly rigorous foundation for the principles of infinitesimal analysis. The statement is so frequently made that the differential calculus deals with continuous magnitude, and yet an explanation of this continuity is nowhere given; even the most rigorous expositions of the differential calculus do not base their proofs upon continuity … they either appeal to geometric notions … or depend upon theorems that which are never established in a purely arithmetic manner.

What are numbers, and what should they be? (1888)

In speaking of arithmetic (algebra, analysis) as a part of logic I mean to imply that I consider the number-concept entirely independent of the notions or intuitions of space and time, that I consider it an immediate result from the laws of thought.

… numbers are free creations of the human mind; they serve as a means of apprehending more easily and more sharply the difference of things. It is only through the purely logical process of building up the science of numbers and thus acquiring the continuous number-domain that we are prepared accurately to investigate our notions of space and time by bringing them into relation with this number-domain created in our mind.

B. Russell: *The Principles of Mathematics* (1903), paragraph 419

There is thus no mystery to the continuity of space, and no need of any notions not definable in Arithmetic.

L.E.J.Brouwer: `The Nature of Geometry'
Collected Works, Vol.1

Must it be concluded that there is no a priori form of perception of at all for the world of experience? There is, but only in so far as any experience is perceived as spatial or non-spatial change, whose intellectual abstraction is the intuition of time or the intuition of two-in-one. From this intuition of time, independent of experience, all the mathematical systems, including spaces with their geometries, have been built up …

D. Hilbert: *Geometry and Imagination* (1932), (Preface).

… the common superstition that mathematics is but a continuation, a further development, of the fine art of arithmetic, of juggling with numbers. Our book aims to combat that superstition, …