ADDING INDISCERNIBLES BY FORCING WITH AN APPLICATION TO THE MODAL LOGIC OF FORCING

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The modal logic of forcing was introduced by Hamkins and Löwe [3], in which the modal operators $\Diamond$ and $\Box$ are interpreted as

$$\Diamond\phi \iff \phi \text{ holds in some set generic extension of the universe}$$

and

$$\Box\phi \iff \phi \text{ holds in all set generic extensions of the universe.}$$

In [3], it is shown that the modal logic of forcing is exactly $S4.2$. One of the key points in their proof is the existence of buttons and switches.

Given a class of forcing notions, Γ, we can define the modal logic of the class Γ in the same way, by restricting the forcing notions to elements of Γ. In [4], among many other things, it is proved that the modal logic of collapse forcing is $S4.3$.

Hamkins and Löwe have asked, in connection with results in [3], whether there can be a model N of ZFC such that $N \equiv N[H]$ whenever H is the generic collapse of any cardinal onto ω. This question appeared later in Hamkinss paper [2], as question 10. The existence of such a model has strange impacts on the modal logic of collapse forcings. So a model of set theory as N above would be an extreme counterexample in having no switches at all for the class of collapse forcing, and would have valid principles of collapse forcing that are beyond S5, a hard upper bound for the other natural classes of forcing.

In this talk, I will review the above mentioned work of Hamkins and Löwe and then, I'll sketch a solution to the Hamkins-Löwe's question. The proof uses large cardinals and is based on adding a closed unbounded class of indiscernibles using forcing.

This is joint work with William Mitchell [1].

References

[1] Golshani, Mohammad; Mitchell, William, On a question of Hamkins and Löwe on the modal logic of collapse forcing, submitted.

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[3] Hamkins, Joel David; Löwe, Benedikt, The modal logic of forcing. Trans. Amer. Math. Soc. 360 (2008), no. 4, 1793-1817.

[4] Hamkins, Joel David; Leibman, George; Löwe, Benedikt, Structural connections between a forcing class and its modal logic. Israel J. Math. 207 (2015), no. 2, 617651.

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